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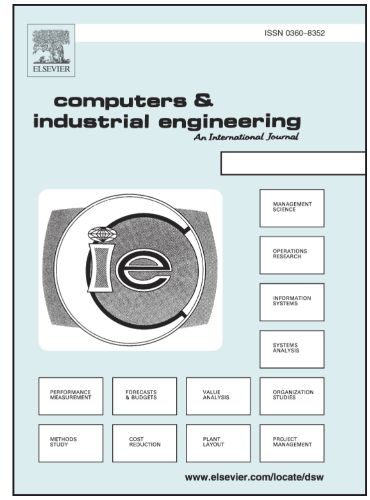

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Retailer Credit Guarantee in a Supply Chain with Capital Constraint under Push & Pull contract

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Highlights

(1) We study how the order contract impact on the financial and operational decision.

(2) We distinguish the order contract in the angle of inventory risk allocation.

(3) The impact of capital constraint level on the financing decision is studied.

(4) We study how banks decide interest rates to keep break-even.

Abstract: We study the credit guarantee scheme used in a supply chain finance (SCF) system including a manufacturer with capital constraint, a retailer and a bank in the competitive credit market. Established a Stackelberg model with the retailer as the leader, we study the operational and financial strategies under different order contracts (push contract and pull contract). Different order contracts result in different inventory risk allocation in supply chain, thus the retailer adopts different variables to influence the manufacturer's decision (push contract: the order quantity; pull contract: credit guarantee level). And we perform a comparative analysis of the optimal strategies among the two scenarios, including a traditional supply chain without SCF as well as one with credit guarantee financing. The results show that the retailer can gain more income while the financing risk is higher under the pull contract. When the manufacturer's capital level is less than a certain threshold, the retailer can provide credit guarantee to bring more benefits to all participants. In addition, a numerical analysis is conducted about the impact of manufacturer's initial capital level and bank inventory supervision cost on the bank's interest rate. The results help the retailer to manage different types of manufacturer, the manufacturer with capital constraint to decide how to finance, and the bank to better control financing risk.

Keywords: push contract; pull contract; capital constraint; credit guarantee; operational and financial decision

1. Introduction

Small and medium-sized enterprises (SMEs) play an important role in promoting economic development. The criteria for the classification of SMEs are based on the industry type, number of employees, sales volume, total assets and other indicators. For industrial enterprises in China, SMEs' employee number is under 1000 or revenue is less than 400 million RMB. As the market competition is fierce and given the rapid development of SMEs, an increasing number of SMEs encounter capital constraint. It is difficult for SMEs to obtain enough loans from banks to preserve optimal operations because they generally have weak credit ratings. This not only results in reducing SMEs' performance but also impacts the performance of upstream and downstream firms along the entire supply chain.

An effective way to solve this problem is to apply supply chain finance (SCF), so that the bank can generate enthusiasm to provide loans to SMEs with low credit. In terms of sharing financial resources, SCF might provide untapped potential for reducing the cost of capital in order to facilitate financing of necessary investments (Randall and Farris, 2009). Some works (Pfohl and Gomm, 2009; Wuttke et al., 2013) show that SCF models do not only aim at reducing costs, but also at mitigating risk. For the actual situation of Chinese SMEs' lack of credit guarantee, the financing method that the core enterprise in the supply chain providing credit guarantee for the SME is common used. For example, Ali's small loan SCF has developed Ali credit loans and other micro credit products. Between 2012 and 2016, Ali offered small loans to SMEs every year. Since 2016, China's SCF market size had exceeded 10 trillion RMB, and it is expected to reach nearly 20 trillion RMB by 2020. The stock market space is huge. Our research focuses on an innovative credit facility initiated by

China Commercial Bank approximately six years ago that is now gaining recognition in the United States. We concentrate on external Bank Credit Financing provided partner loans, which is one source of SCF (Yan N, 2016). We explore a distribution channel including a manufacturer with capital constraint and a retailer, in which the retailer is the leader.

We make this study because there exists a real-world example in which a Chinese commercial bank (Industrial and Commercial Bank of China, ICBC) provides loans to support SMEs in consideration of credit guarantee. This representative case focuses on ensuring merchandise quality and a stable supply for Walmart in China in its purchase of commodities from a large supplier at home. Further, Walmart doesn't have enough cash to pay for its purchase, for example, it is reported that Walmart had defaulted the account of the manufacturer Qian Qian Sheng about 1.5 million RMB from July 2012 to April 2016 (http://www.sohu.com/a/69565163_112101). And the manufacturers encountered loan trouble because of their weak credit ratings. However, with a credit guarantee from the downstream partner and dominating retailer Walmart, ICBC offered the manufacturers loans. In other words, the retailer committed to repay some portion of the loans if the manufacturer faces bankruptcy. In this way, the ICBC Bank can relax its screening and monitoring process when it extends loans to the company, which then increases liquidity and enables the company to complete the order. Consequently, all companies are happy to accept the credit guarantee from the retailer. Another example of this type of SCF is JingDong Co. Ltd (NASDAQ: JD). This firm also lacked the cash to pay its manufacturers because it only offered cash on delivery. JD cooperated with PingAn bank to offer a credit guarantee to their upstream partner. This approach can effectively protect the lender's profit from the borrower's default.

As a retailer expands in size, it may engage with different types of manufacturers. Some large retailers or companies, such as Walmart, JingDong, have a large number types of supplier, and the order modes vary because the product type and market demand are different, which generate diverse SCF modes. The terms of trade between the firms are chosen from two types of order contracts, that is, push contract and pull contract. Under the push contract, the retailer must pre-order products from the manufacturer before the sale season, and the manufacturer only produces the retailer's order quantity. In this way, the retailer will bear the inventory risk in the supply chain. Under the pull contract, the retailer will not order from the manufacturer in advance until the sales season arrives. Instead, the retailer only places orders with the manufacturer when actual market demand arises during the sales season. Thus the inventory is maintained by the manufacturer, and the manufacturer takes all the inventory risks. Besides, according to Cachon (2004), there are two other situations that can be represented by a pull contract: Vendor Managed Inventory or drop shipping (the supplier holds the inventory and ships directly to consumers, bypassing the retailer). The manufacturer's delivery cycle, the amount of capital tied up and the lead time is different under different contracts. At this point, the retailer's decisions revolve around financing demands in the two modes and providing credit guarantees for the manufacturers. This will not only impact the retailer's profit but also can affect all participants' revenues.

This paper studies how the retailer's order contract mode influences the supply chain's financial and operational decision, based on the retailer's credit guarantee to the manufacturer with capital constraint. We study separately under the two different kinds of financing scenarios: a traditional supply chain without SCF and with SCF (credit guarantee), and the conditions for the supply chain members to participate in SCF are also obtained. The results can provide financing and ordering decision supports for retailers with multiple manufacturers. It can also help manufacturers with financial constraints decide whether to undertake credit guarantee financing with different order contract. Since the problem of capital constraint is studied, it is also worth considering how the manufacturer's initial capital level affects the supply chain's performance and the member's profit distribution. The retailer bears bigger inventory risk under a push contract than pull contract. Thus, the retailer will consider different decision variables for different contracts to coordinate the manufacturer's and the bank's decisions. In this case, the supply chain decision model is different under the two order contracts. For the bank in the competitive credit market, it will keep break-even and control risk by setting loan interest rate. And all the supply chain member's decision will impact on the bank's decision. So the further numerical analysis is conducted on the impact of the manufacturer's initial capital level, the retailer's credit guarantee level and the inventory supervision cost on the bank's interest rate.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the basic SCF model, including the model's framework, notations and assumptions. Section 4 discusses the SCF equilibrium under push and pull contracts separately. Section 5 presents numeral experiment by Matlab and discusses the results of our analysis. In Section 6 this paper provides concluding remarks as well as existing limitations, and discusses some directions for future work. All proofs are provided in the Appendix.

2. Literature review

Our work is related to three branches of literature. The first stream addresses the supply chain with capital constraint (retailer with capital constraint, manufacturer with capital constraint, and both with capital constraint). The second stream is about the credit guarantee financing mode. The third stream is about the supply chain contract literature with a focus on push contract and pull contract.

2.1. Supply chain with capital constraint

The first stream considers a supply chain with capital constraint. Considering a manufacturer with capital constraint in a two-echelon supply chain, Xu and Birge (2004) explain how capital constraint may affect a firm's production decisions. They conclude that companies with internal capital constraint can improve performance by appropriately considering debt along with production quantities. They also show that both production and financial decisions are related to capital structure variables. Dada and Hu (2008) analyze a retailer with capital constraint borrowing funds from the bank for procurement. They showed that if the borrowing cost is low, the retailer borrows to procure a quantity that is less than the channel's optimal quantity. In this way, the bank charges an interest rate that decreases based on the equity position of the firm. Raghavan and Mishra (2011) consider one retailer and one manufacturer, where both firms are experiencing financial problem and can't achieve their optimal quantity. They find that if the cash of one firm in the supply chain is very low, the supply chain financing decision can be better for both the lender and the whole supply chain. Kouvelis and Zhao (2016) study contract design and coordination in a two-echelon supply chain facing uncertain demand. All the supply chain members face capital constraint and need to borrow competitively priced bank loans for their operational requirements.

It can be seen that capital constraint is a common problem in the supply chain operation decision. And based on the fact that there exist many supply chains that contain strong retailer and weak SME manufacturer, it is necessary to study the impact of manufacturers' capital constraint on supply chain decision making in this paper.

2.2. Credit guarantee financing

The second stream reviews the research related to credit guarantee. Credit guarantee is a financial support mode commonly adopted by countries all over the world to solve the financing difficulties of SMEs. Su and Zhong (2017) consider the core enterprise credit guarantee financing model and calculate the expected profit function of each member in supply chain. Based on the Stackelberg game model, the optimal decision of each member in the decentralized system and the centralized system are given. The results show that under the credit guarantee of the core enterprise, the retailer has the best ordering strategy, and the wholesale price of the core enterprise is the best. Yan et al. (2017) study the impact of financial risk aversion and decision preference on equilibrium between bank credit and credit guarantee. A mathematical analysis is made for the optimal ordering quantity, wholesale price and bank interest rate under different risk preference scenarios. The results show that combining the bank credit with the credit guarantee can effectively balance the risk of the retailer's financing between the bank and the manufacturer through the rate of interest rate and the wholesale price. Marchi et al. (2016) consider the case where a buyer offers a credit to its vendor, in purpose to develop the vendor by improving its production capabilities and subsequently its performance.

It shows that limited access to capital and the linked worst credit condition at the vendor's side may lead to inferior solutions for the supply chain, and a partnership agreement on sharing financial resources between supply chain members may help to overcome the potential skepticism of uncertain investments. To alleviate the SMEs' financing difficulties by establishing the credit guarantee organization, which is the prevailing practice of the countries supporting the development of SMEs. In the supply chain, the retailer as the core enterprise plays a role as the credit guarantee organization. Credit guarantee is a breach of the SMEs' financing, and responsibility sharing among banks, credit-guarantee organization and SMEs is key for credit guarantee.

2.3. Push and pull contracts

The third stream is the supply chain contracting literature with a focus on push and pull contracts. Cachon (2004) studies how the allocation of inventory risk impacts on supply chain efficiency, and found that the efficiency of a wholesale price contract is higher than previously thought if firms adopt both push and pull contracts. Budde (2014) investigates a retailer sourcing problem where the retailer has the option to source from multiple suppliers. They found that only pull contracts lead to supply chain coordination. Gou et al. (2016) study push and pull contracts in a local supply chain. The results show that the supplier's production capacity plays an important role; it affects the supplier's negotiating power with the retailer, and coordination of the supply chain can be achieved only with enough capacity. Davis (2012) investigates how the allocation of inventory risk in a two-stage supply chain affects channel efficiency under push contract and pull contract. The results show that a pull contract achieves higher channel efficiency than a push contract. The push contract and pull contract are two common order patterns in the supply chain, and the inventory risk distribution in the supply chain is different under the two contracts. Thus the retailer's financing guarantee decision and the manufacturer's production and pricing decision are also different. The impact of the change of order models on supply chain decisions is the problem to be explored in this paper.

The most relevant literature with this paper are Cachon (2004) and Yan et al. (2016). Cachon (2004) studies how the allocation of inventory risk (via push contract, pull contract and Advance-Purchase Discount contract) impacts supply chain efficiency. The push and pull contract put forward by Cachon (2004) provides a good reference for this paper. The difference is that Cachon (2004) focuses on the efficiency of the supply chain under the three contracts, while this paper focuses on the optimal decision of the supply chain members. Yan et al. (2016) study the strategy of supply chain financing equilibrium and coordination with capital constraint based on partial credit guarantee contract, indicating that core enterprises provide credit guarantee for SMEs, which is an effective way to solve the financing difficulties of SMEs. So credit guarantee is adopted in this paper as the research object. Yan et al. (2016) perform a comparative analysis of the optimal strategies among the various financing scenarios, including a traditional supply chain without capital constraint and SCF without credit guarantee as well as with full credit guarantee, which is an enlightening significance for this paper. Unlike Yan et al. (2016), this paper regards the retailer as the core enterprise in the supply chain, and the manufacturer is likely to face capital constraints. Moreover, we consider two different ordering models. Under these two modes, the inventory risk allocation of supply chain is different, and the decision parameters have also changed. Under the two different kinds of financing scenarios (a traditional supply chain without financing and SCF with credit guarantee), the optimal financing and operation decision of supply chain under push and pull contract is compared, so the retailer can make better choice of order contracts under different financing scenarios. Furthermore, we get the conditions for the supply chain members to participate in financing.

3. Model

The supply chain consists of a manufacturer (referred to as “she”), a retailer (referred to as “he”) and a commercial bank. The retailer purchases items from the manufacturer with capital constraint under a push contract or a pull contract and sells to the final market; the retailer doesn’t know the market demand for the product when he purchases before the sale season. And there exists a problem that the manufacturer is with capital constraint in the production process and cannot meet the needs of the retailer’s optimal order only using her initial capital. To avoid supply chain disruption and gain more profit, the manufacturer decides to obtain financing from the bank. Because the manufacturer has low credit worthiness, it isn’t easy to obtain a bank loan; this can be made much easier if the retailer provides a credit guarantee for the loan. With the guarantee, the retailer would repay part of the loan if the manufacturer goes bankrupt, thereby reducing the default risk for the bank. In practice, a bank will entrust third party logistics enterprises with supervision over products produced by manufacturers to control risk; these do not deliver the goods until the retailer pays for the order. Fig.1 shows the supply chain financing framework.

Fig.1. the supply chain financing framework

The manufacturer obtains loans from the bank with the retailer’s credit guarantee. Then the manufacturer uses the capital for production and transfers the products to the warehouse commissioned by the bank. The retailer will pay for the products to the special account in the bank, and then the product will be delivered from the warehouse. After deducting the interest and the principal, the bank transfers the revenue to the manufacturer. If there is a lack of funds to repay the loan, the retailer needs to pay the remainder.

3.1. Notations and assumptions

The retailer and the manufacturer are all risk neutral. And as they have established a long-term partnership, this paper does not consider the situation of information asymmetry, that is, they can verify the market demand distribution, the manufacturer’s initial capital level and other information (Kouvelis & Zhao, 2012). The manufacturer’s product cost is C, initial capital is K_m. The retailer orders Q from the manufacturer, then the manufacturer produces the quantity q. The product is delivered to the retailer at the wholesale price w, then the retailer sells to the customer at the retail price p.

If the manufacturer faces capital constraints and requires loans, with the retailer’s credit guarantee level of L, the manufacturer can get loans of L at most. The commercial bank's risk-free interest rate is R_f, while the bank charges the manufacturer's loan at the interest rate R_r, $R_r \geq R_f$. The fee charged by the bank for the supervision of the third party logistics company is h. $C(1+R_r) < w < p$, $h < C$ and there is no salvage value for the unsold product. The goodwill loss because of stock out isn't considered.

In addition, $\pi_i (i = m, r, b, s)$ denote the expected profit, and the subscript m, r, b, s refer to the manufacturer, retailer, bank, and the whole supply chain, respectively. Let Q^*, w^*, L^*, q^* and R_r^* be the optimal order quantity, wholesale price, credit guarantee level, production output and interest rate, respectively.

Market demand D is randomly distributed, $F(D)$ and $f(D)$ represent the demand distribution function and probability density function respectively. The market demand satisfies the strictly Increasing Generalized Failure Rate (IGFR), that is, $Dh(D) = \frac{Df(D)}{1-F(D)}$ increases with D, ensuring the existence and uniqueness of the solution. Many common distributions, such as uniform distribution, normal distribution, exponential distribution and so on meet the IGFR condition (see Lariviere and Porteus, 2011; Cai et al., 2014; Yan et al., 2016). Besides, the capital market is assumed to be competitive and the bank's profit is zero, i.e. $\pi_b = 0$. It means that the financing service provided by the bank will meet the expected return which is on the risk free interest rate capital market. It is common in an SCF with a fairly priced loan.

3.2. Sequence of events under the push contract

Under the push contract, the retailer pre-orders product from the manufacturer before the sale season, thus he holds the inventory and may face inventory risk. The retailer as the Stackelberg leader first determines the optimal order quantity Q, then the manufacturer decides the optimal wholesale price w. In reality, refer to Zhong (2008), the retailer first releases his potential maximal order quantity Q^M and then determines the final order quantity Q according to the manufacturer's wholesale price w, that is, $Q = Q^M - w\beta$, $\beta > 0$, where β represents sensitive coefficient of the retailer orders to the wholesale price. If the manufacturer does not have enough cash to fulfil the order, she needs to decide whether to apply for a loan from the bank. The bank will charge the loan at the interest rate R_r and the loan quantity is $L = CQ - K_m$. After the realization of natural demand, the manufacturer should repay the bank loans with her income or declare bankruptcy if she cannot repay them. When the manufacturer goes bankrupt, the retailer will pay for the remainder loans with the limit of credit guarantee. Fig.2 illustrates the timeline of events.

Fig. 2. Sequence of events under the push contract

3.3. Sequence of events under the pull contract

Under the pull contract, the retailer won't order product from the manufacturer until the sale season arrives, thus the manufacturer holds the inventory and may face inventory risk. In order to support the manufacturer with capital constraint to produce more product, the retailer first declares his credit guarantee level L, then the manufacturer determines the production quantity q. The retailer will place an order to the manufacturer after the demand is realized, so the order quantity will be $\min\{q, D\}$. The bank will charge the loan at the interest rate R_r. After the realization of natural demand, the manufacturer should repay the bank loans with her income or declare bankruptcy if she cannot repay them. When the manufacturer goes bankrupt, the retailer will pay for the remainder loans with the limit of credit guarantee. Fig.3 illustrates the timeline of events.

Fig. 3. Sequence of events under the pull contract

4. Analysis

This section mainly studies the optimal decision of supply chain members under the push contract and the pull contract separately, under the two different kinds of financing scenarios: a capital constrained supply chain without SCF and with the retailer's credit guarantee financing. As the degree of the manufacturer's capital constraint will affect the decision of the members of the supply chain, this paper further explores the influence of the manufacturer's initial capital level on the retailer and the manufacturer's decision-making on participation in SCF.

4.1. Equilibrium results under the push contract

(1) The bank's break-even condition

We assume that the bank is in a competitive credit market, the bank plays the role of funds provider, so that the expected profit from providing a loan should be equal to the funds invested in the capital market at the risk-free interest rate. The bank's expected profit from the loan to the risk-free interest rate market and the manufacturer with capital constraint is given by $E[\pi_{\mathrm{b0}}]$, $E[\pi_{\mathrm{b1}}]$, then

$$E[\pi_{\mathrm{b0}}] = LR_f$$

$$E[\pi_{\mathrm{b1}}] = \min\{L(1+R_{\mathrm{r}}), wQ\} - L - Qh$$

According to the description and hypothesis, we put $E[\pi_{\mathrm{b0}}] = E[\pi_{\mathrm{b1}}]$, hence, the bank's break-even condition can be formulated by Eq. (1)

$$\min\{L(1+R_{\mathrm{r}}), wQ\} - L - Qh = LR_f \tag{1}$$

Therefore, the bank's interest rate is related to the amount of loan (i.e. the guarantee amount of the retailer), the expected revenue from the manufacturer and the regulatory cost that the bank has to pay.

(2) The retailer's profit

The retailer is the leader of the Stackelberg model, so he first makes the decision. At first, the retailer orders a quantity Q from the manufacturer, then the manufacturer makes wholesale at w. The manufacturer then makes a loan L from the bank with an interest rate R_{r}, where $L = CQ - K_m$. After the demand is realized, the retailer obtains $p\min\{Q, D\}$. He earns the profit $p\min\{Q, D\} - \mathrm{w}Q$ if the manufacturer can repay the loan. Otherwise, the retailer must repay the remaining part as $L(1+R_{\mathrm{r}}) - wQ$. So the order quantity model with the credit guarantee can be formulated by Eq. (2). The retailer's expected profit is given by $E[\pi_{\mathrm{r1}}]$ when participating in SCF.

$$E[\pi_{\mathrm{r1}}] = p\min\{Q, D\} - \mathrm{w}Q - E[L(1+R_{\mathrm{r}}) - wQ]^{+} \tag{2}$$

Where $E[L(1+R_{\mathrm{r}})-wQ]^{+}=\begin{cases} L(1+R_{\mathrm{r}})-wQ & if L(1+R_{\mathrm{r}})\geq wQ \\ 0 & if L(1+R_{\mathrm{r}})<wQ \end{cases}$.

If the retailer refuses to offer credit guarantee for the manufacturer, or the manufacturer is unwilling to participate in financing, the retailer's expected profit $E[\pi_{\mathrm{r}0}]$ is given by Eq. (3).

$$E[\pi_{\mathrm{r}0}]=p\min\{Q,D\}-\mathrm{w}Q,(Q\leq\frac{K_m}{C}) \tag{3}$$

(3) The manufacturer's profit

The manufacturer is the follower of the Stackelberg model. The manufacturer receives a revenue wQ after the demand is realized. If her revenue is sufficiently high, the manufacturer will repay $L(1+R_r)$ to the bank. Otherwise, the manufacturer goes bankrupt and has to repay all of her revenue wQ to the bank. The pricing decision model with the credit guarantee can be described in Eq. (3). Let $E[\pi_{m1}]$ denote the manufacturer's expected profit under SCF; $E[\pi_{m1}]$ is given by Eq. (4).

$$E[\pi_{m1}]=E[wQ-L(1+R_r)-K_m]^{+} \tag{4}$$

Where $E[wQ-L(1+R_r)-K_m]^{+}=\begin{cases} wQ-L(1+R_r)-K_m & if wQ\geq L(1+R_{\mathrm{r}})+K_m \\ 0 & if wQ<L(1+R_{\mathrm{r}})+K_m \end{cases}$.

If the manufacturer doesn't participate in financing, then her profit $E[\pi_{m0}]$ is given by Eq. (5).

$$E[\pi_{m0}]=(w-C)Q,(Q\leq\frac{K_m}{C}) \tag{5}$$

Proposition 1. *Assume that the manufacturer doesn't participate in SCF. The retailer's optimal order quantity* Q^* *and the manufacturer's optimal wholesale price* w^* *are given by:* $Q^*=\frac{K_m}{C}$, $w^*=\frac{K_m}{C\beta}+C$.

Proposition 1 suggests that the retailer's optimal order quantity in the supply chain where exists capital constraint without financing services only depends on the operational parameters, namely, the manufacturer's initial capital level and the product cost. For the manufacturer, the optimal wholesale price relies on her initial capital and the coefficient capturing the sensitivity of the orders to the wholesale price.

The optimal wholesale price increases in the manufacturer's initial capital level increases and decreases in the sensitivity coefficient increases.

It is obvious that when the manufacturer with capital constraint obtains SCF service under the push contract, the bank is borrowing without bankruptcy risk, and the retailer will not take the risk of paying the guarantee fee. In this case, the bank and the retailer have a willingness to participate in SCF.

Proposition 2. *Assume that the manufacturer participates in SCF. Under the retailer's credit guarantee and the push contract, the manufacturer will certainly be able to repay the bank loan and the corresponding interest after the selling season. The optimal wholesale price* w^**, the optimal order quantity* Q^* *and the bank's optimal loan risk interest rate* R_r^* *satisfy*

$$Q^* = \max(Q_1, 0),$$

$$w^* = \frac{Q^*}{\beta} + C(1+R_f) + h,$$

$$R_r^* = R_f + \frac{Q^* h}{CQ^* - K_{\mathrm{m}}},$$

Where Q_1 *satisfies* $Q_1 = \dfrac{p\bar{F}(Q_1) - C(1+R_f) - h}{\beta+1}\beta$.

From Proposition 2, it is obvious that when the manufacturer with capital constraint obtains SCF service under the push contract, the bank borrows without bankruptcy risk, and therefore, the retailer will not take the risk of paying the guarantee fee. In this case, the bank and the retailer have a greater willingness to participate in SCF. The proposition supposes that the retailer has a higher decision-making power by acting as the Stackelberg leader. The retailer's order quantity decision will have an impact on the manufacturer and the bank's decisions. When the order quantity is high, correspondingly, the credit guarantee level will be high; in this case, the bank sets a low loan interest rate risk, as this will meet the requirements for it to benefit. Conversely, the wholesale price is high, and the retailer will bear stock risk. Thus, in order to maximize his profit, a rational retailer will meet a trade-off and choose the appropriate order quantity.

Proposition 3. *Under a push contract, the manufacturer and the retailer are willing to participate in SCF when the manufacturer's initial capital level is below a certain* *threshold. When the market demand follows a uniform distribution with* b *as an upper bound and* a *as a lower bound, this threshold satisfies* $\min(K_{m1}, K_{m2})$

$$K_{m1}=\frac{M_0-\sqrt{M_1Q^{*2}+M_2Q^*+M_3}}{2(b-a)+p\beta},$$

$$K_{m2}=\frac{C^2\beta R_f-C\sqrt{C^2\beta^2R_f^2+4Q^{*2}}}{2},$$

Where $M_0=[(p-C)b+aC]\beta C$,

$M_1=[2(b-a)+p\beta](p+b-a)C^2$,

$M_2=-2[2(b-a)+p\beta][pb-(b-a)(C+CR_f+h)]\beta C^2$,

$M_3=[(C-p)b-aC]^2C\beta^2$,

$$Q^*=\frac{pb\beta-\beta(b-a)(C+CR_f+h)}{(1+\beta)(b-a)+p\beta}.$$

Proposition 3 shows that only when the manufacturer's initial capital level is in a certain range, the retailer is willing to provide credit guarantee, and the manufacturer is willing to participate in SCF. When the manufacturer's initial capital level is very low, the guarantee amount that the retailer should provide is very high, he will bear high inventory risk and credit guarantee risk under the push contract. When the manufacturer's initial capital level is relatively high, the cost of manufacturer's loan is greater than the earnings acquire from producing more products, so the manufacturer will not choose SCF.

4.2. Equilibrium results under the pull contract

(1) The bank's break-even condition

The bank is in a competitive credit market, in which the expected return from providing a loan should be equal to the funds invested in the capital market at the risk-free interest rate. The bank's expected profit from the risk-free interest rate market and the capital-constrained manufacturer is given by $E[\pi_{\mathrm{b2}}]$, $E[\pi_{\mathrm{b3}}]$ then

$$E[\pi_{\mathrm{b2}}]=LR_f$$

$$E[\pi_{\mathrm{b3}}]=LR_{\mathrm{r}}-qh$$

According to the description and hypothesis, we put $E[\pi_{b2}] = E[\pi_{b3}]$, hence, the bank's optimal endogenous interest rate problem can be formulated as Eq. (6).

$$LR_r - qh = LR_f \tag{6}$$

Therefore, the bank's interest rate is related to the amount of loan (i.e. the guarantee amount of the retailer), the manufacturer's production output and the regulatory cost that the bank has to pay.

(2) The retailer's profit

The retailer is the leader of the Stackelberg model, so he first makes the decision. The retailer first provides a credit guarantee for his manufacturer, then the manufacturer decides the production quantity. Under the pull contract, the retailer orders product according to market demand and the manufacturer's production quantity, and the order quantity is the smaller of these, that is, $\min\{q, D\}$.The retailer gets the profit $(p-w)\min\{q, D\}$ if the manufacturer's return is sufficient to repay the bank loan; otherwise, the retailer must undertake the remaining debt as $L(1+R_r) - w\min\{q, D\}$. Hence, the retailer's credit guarantee level decision model with the credit guarantee can be formulated as Eq. (7). Assuming that the retailer's profit is $E[\pi_{r3}]$ when participating in SCF.

$$E[\pi_{r3}] = (p-w)\min\{q, D\} - E[L(1+R_r) - w\min\{q, D\}]^+ \tag{7}$$

Where $E[L(1+R_r) - w\min\{q, D\}]^+ = \begin{cases} L(1+R_r) - w\min\{q, D\} & if L(1+R_r) \geq w\min\{q, D\} \\ 0 & if L(1+R_r) < w\min\{q, D\} \end{cases}$.

If the retailer refuses to offer credit guarantee for the manufacturer, or the manufacturer is unwilling to participate in SCF, then the retailer's expected profit $E[\pi_{r2}]$ is given by Eq. (8).

$$E[\pi_{r2}] = (p-w)\min\{q, D\}, (q \leq \frac{K_m}{C}) \tag{8}$$

(3) The manufacturer's profit

Under the pull contract, $Q = \min\{q, D\}$. The manufacturer faces uncertain demand, and the maximum production output she can achieve using only her initial capital is far less than the average market demand. We regard the product wholesale price w as exogenous, which occurs in several real-life situations: the procurement of some products that depend on scarce resources is price-governed by the government; if the retailer and the manufacturer sign a long-term contract, we can say that the wholesale price is exogenous over a long period of time; it is also exogenous when wholesale price formulation and production decisions belong to different departments in enterprises (refer to Zer, 2006, and Dong, 2007).

The bank provides loans on the basis of the retailer's loan guarantee level L, and the risk interest rate is R_r. The manufacturer has a minimum output if she wants to participate in SCF. The manufacturer can repay the bank loan if demand is greater than the output $\frac{L(1+R_r)}{\mathrm{w}}$; otherwise the manufacturer is unable to repay the bank loan, and she must use all of her revenue wQ to repay the bank, while the retailer repays the remainder loan. Let $E[\pi_{m3}]$ denote the manufacturer's profit, and $E[\pi_{m3}]$ is given by Eq. (9).

$$E[\pi_{\mathrm{m}3}] = E[w\min(q,D) - LR_r - Cq]^+ \tag{9}$$

If the manufacturer doesn't participate in SCF, then her profit $E[\pi_{m2}]$ is given by Eq. (10).

$$E[\pi_{\mathrm{m}2}] = w\min(q,D) - Cq, (q \le \frac{K_{\mathrm{m}}}{C}) \tag{10}$$

Proposition 4. *Assume that the manufacturer doesn't participate in SCF. The optimal order quantity* Q^* *and the optimal production quantity* q^* *are given by:*

$$Q^* = \min\{\frac{K_m}{C}, D\},$$

$$q^* = \frac{K_m}{C}.$$

It is obvious that when the demand is too low, it may result in low revenues. And the manufacturer can't repay the bank and would go bankrupt. The bankruptcy risk would transfer to the retailer because of the retailer's credit guarantee. Thus, a rational retailer will choose a suitable credit guarantee level to affect the manufacturer's decision and would then control the risk of bankruptcy to avoid bearing joint liability.

Proposition 5. *Assume that the manufacturer doesn't participate in SCF. Under the retailer's credit guarantee and a pull contract, the sufficient condition for the manufacturer to assume the loan without bankruptcy risk is the demand that satisfies* $\hat{D}_1 = \frac{L(1+R_r)}{w}$. *The optimal production quantity* q^* *, the optimal credit guarantee level* L^* *and the bank's optimal risk interest rate* R_r^* *satisfy*

$$L^* = \min\{CF^{-1}(1 - \frac{C + hF(\hat{D}_1)}{w}) - K_m, L_0\},$$

$$q^* = \frac{L^* + K_m}{C},$$

$$R_r^* = R_f + \frac{q^* h}{L^*},$$

Where L_0 *satisfies* $\frac{p-w}{C}\bar{F}(\frac{L_0 + K_m}{C}) = (1 + R_r^* - \frac{hK_m}{CL_0})F(\hat{D}_1)$.

From Proposition 5, the retailer's optimal credit guarantee level will impact the manufacturer's and the bank's decisions. Because the retailer takes some risk for providing the credit guarantee, his credit guarantee level will not exceed the amount that the manufacturer needs for production. The optimal credit guarantee level will exactly meet the optimal production output. The interest rate decreases with the credit guarantee level increased, and then the risk of failing to recover the loan will be reduced.

Proposition 6. *Under a pull contract, the manufacturer and the retailer will participate in SCF when the manufacturer's initial capital level is below a certain threshold. When the market demand distribution follows a uniform distribution with an upper bound of* b *and a lower bound of* a*, this threshold satisfies* $\min\{K_{m3}, K_{m4}\}$*, where* K_{m3} *satisfies* $\min\{K_1, K_2\}$, K_{m4} satisfies $\min\{K_3, K_4\}$.

K_1 is the root of $A_1K^2 + B_1K + C_1 = 0$; K_2 is the root of $A_2K^2 + B_2K + C_2 = 0$;

K_3 is the root of $A_3K^2 + B_3K + C_3 = 0$; K_4 is the root of $A_4K^2 + B_4K + C_4 = 0$;

where $A_1 = (2wCNX_2 - 1)h^2 + (NC - 1)wX_2^2$,

$B_1 = -2wC(NX_1 + MX_2) - 2wX_1X_2(NC-1)$, $C_1 = 2wC(NX_1^2 + MX_1 + ah) - wX_1^2$;

$A_2 = (2wCNX_4 - 1)h^2 + (NC-1)wX_4^2$,

$B_2 = -2wC(NX_3 + MX_4) - 2wX_3X_4(NC-1)$, $C_2 = 2wC(NX_3^2 + MX_3 + ah) - wX_3^2$;

$A_3 = (N^2+1)w^2X_2^2 + 2(Nh-w)wX_2 + h^2$,

$B_3 = 2(N-1)bCw^2X_2 - 2(N^2+1)w^2X_1X_2 - 2(Nh-w)wX_1 - 2bChw$,

$C_3 = 2(1-N)bCw^2X_1 + (N^2+1)w^2X_1^2$;

$A_4 = (N^2+1)w^2X_4^2 + 2(Nh-w)wX_4 + h^2$,

$B_4 = 2(N-1)bCw^2X_4 - 2(N^2+1)w^2X_3X_4 - 2(Nh-w)wX_3 - 2bChw$,

$C_4 = 2(1-N)bCw^2X_3 + (N^2+1)w^2X_3^2$;

$N = \dfrac{C(1+R_f)+h}{w}$, $M = (p-w)b + ac(1+R_f) + ah$,

$X_1 = \dfrac{Cw[(w-C)b + a(C+h)]}{w^2 + h^2 + C(1+R_f)h}$, $X_2 = \dfrac{Cw^2 + h^2}{w^2 + h^2 + C(1+R_f)h}$,

$X_3 = \dfrac{MC}{p - w + N^2w^2}$, $X_4 = \dfrac{Nwh^2}{p - w + N^2w^2}$.

Proposition 6 shows the manufacturer's initial capital level must meet certain conditions to enable the retailer and the manufacturer to participate in SCF. When the manufacturer's initial capital level is very low, the cost of retailer's guarantee is higher than that of reduced inventory holding cost under the push contract. When the manufacturer's initial capital level is relatively high, it's uneconomical for the manufacturer to finance from the bank to prepare too much inventory, which only increases inventory risk and financing costs.

In general, when the manufacturer with capital constraint does not participate in supply chain financing service, the optimal production output under both the push contract and the pull contract are $\dfrac{K_m}{C}$, that is, the manufacturer will use all her capital for production. Obviously, in this case, the retailer has more expected revenue under the pull contract, because he completely transfers the inventory risk to the manufacturer. When the manufacturer with capital constraint participates in SCF, the supply chain members' decision is related to many exogenous variables, such as the manufacturer's initial capital level. As a leader in the supply chain, the retailer affects the manufacturer's decisions in different ways under the two contracts. Under the push contract, the retailer decides the order quantity; while under the pull contract, the retailer's decision is about credit guarantee level. In addition, under the two contracts, whether the supply chain members participate in SCF or not depends on the manufacturer's capital level.

5. Numerical analysis

In order to further explore the impact of the manufacturer's initial capital level on the supply chain's profit, and how the retailer's decision and the manufacturer's capital status affect the bank's financing interest rate together, both under the push contract and the pull contract, this section carries out some numerical experiments to visualize the research results intuitively. Besides, sensitivity analysis about the bank's stock supervision cost to the bank's decision is conducted.

In light of the distribution and parameters set by Yan (2016), we denote that the market demand distribution function is the uniform distribution function of $D[20,200]$; then $f(D)=\frac{1}{180}$ and $F(D)=\int_{20}^{D}\frac{1}{180}dD$. Denote the unit retail price is $p=120$, the unit product cost is $C=60$, the exogenous wholesale price is $w=80$, the unit bank's inventory supervision cost is $h=10$, the sensitivity coefficient of the order to the wholesale price is $\beta=100$, the manufacturer's initial capital is $K_m \in (0,6000)$, and the risk-free interest rate is $R_f=0.05$.

5.1. Impact of the manufacturer's initial capital level

Fig. 4 and Fig. 5 show the supply chain financing value based on different manufacturer's initial capital levels for the retailer and the manufacturer under the push contract and the pull contract separately.

Fig.4 Impact of the manufacturer's initial capital on the retailer's profit

Fig.4 shows that when the manufacturer's initial capital level is below a certain threshold, it is beneficial for the retailer to provide credit guarantee to the manufacturer to participate in SCF under the two contracts. However, when the retailer participates in SCF, under the push contract, his profit remains unchanged as the manufacturer's initial capital level increases, it means that the initial capital level doesn't affect his profit; under the push contract, his profit increases first and then reduces. And it shows that although the retailer's profit fluctuates under the pull contract, the overall profit is greater than that under the push contract when financing.

Fig.5 Impact of the manufacturer's initial capital on the manufacturer's profit

Fig.5 shows that when the manufacturer's initial capital level is below a certain threshold, participating in SCF is a better choice for the manufacturer. The range of the manufacturer's initial capital level to participate in financing under the push contract is larger than the pull contract, and the profit of the manufacturer is larger under the push contract. It is reasonable that the manufacturer will bear all the inventory risk under the pull contract, so her profit is lost compared to the push contract, and she isn't willing to increase her financing cost when her funds exceed a certain value.

5.2. Impact of the retailer's decision and the manufacturer's capital status

Fig.6 Impact of the retailer's decision and the manufacturer's capital status on the bank's interest rate

Fig.6 describes the impact of the retailer's decision and the manufacturer's capital status on the bank's interest rate. To facilitate the bank's decision, although the retailer's decision variables are the order quantity under the push contract and the credit guarantee level under the pull contract, we can transform the order quantity into the credit guarantee level for the push contract because they have a certain connection. Therefore, this paper sets the retailer's decision variable as the credit guarantee level in the numerical analysis.

It suggests that the connection between the credit guarantee level and the bank's loan risk interest rate is non-linear and that the loan risk interest rate decreases with the retailer's credit guarantee level increased, as the bank's financing risk is reduced with the retailer's credit guarantee. In addition, the change of the manufacturer's initial capital level has a strong effect on this trend. And the amplitude of variation for the bank's interest rate is smaller under the push contract than the pull contract, because the bank faces less financing risk under the push contract.

5.3. Impact of the bank's stock supervision cost

Fig.7 shows the impact of the bank's stock supervision cost h on the bank's interest rate. The commercial bank should meet the conditions of break even, and the loan interest rate reflects the financing risk to some extent.

Fig.7 Impact of the bank's stock supervision cost on the bank's interest rate

The bank's interest rate increases with the bank's stock supervision cost increases, because the bank's overall cost increases. In order to control the risk of financing and avoid malicious default of the SME manufacturer, the bank entrusts the third party logistics companies to supervise inventory, but if the supervision cost is too high, the bank can only increase interest rate to ensure income, or even does not provide financing. It can be seen that the bank's interest rate change is more intense under the pull contract than the push contract, which illustrates that bank faces greater financing risk under the pull contract.

6. Conclusions

This paper studied a supply chain with a SME manufacturer and a retailer with good credit condition. The production lead time is long and the manufacturer has only one production opportunity before the selling season. The retailer can place orders to the manufacturer under the push contract and the pull contract. Accordingly, the allocation of inventory risk in the supply chain is different. This paper focused on the SCF mode that the retailer provides credit guarantee to the manufacturer faced with capital constraint to obtain bank loans. Based on this retailer's credit guarantee financing mode, this paper studied the optimal operational and financial decision of the retailer and manufacturer under the two different order contracts, and made a comparative analysis with two scenarios, including no SCF services and with full credit guarantee.

The main conclusions of the paper are as follows. First, when the manufacturer with capital constraint does not participate in SCF, the manufacturer will use all capital for production under both the push contract and the pull contract. Obviously, in this case, compared to the push contract, the retailer has more expected revenue under the pull contract, because he completely transfers the inventory risk to the manufacturer.

Second, both under a push contract and a pull contract, the manufacturer with capital constraint participates in SCF with the retailer's credit guarantee, which can bring more benefits to the participants and enhance the entire performance significantly. In addition, the retailer obtains more profits as a leader, and he affects the manufacturer’s decisions in different ways under the two contracts (push contract: the order quantity; pull contract: credit guarantee level).

Third, the manufacturer’s initial capital level will affect the supply chain member’s participation of SCF service and their profit. When the level is below a threshold, the retailer and the manufacturer will be willing to participate in SCF. In addition, we find that the manufacturer’s initial capital level threshold to satisfy the two’s willingness to participate in the SCF services will differ. The capital level threshold of the retailer is smaller than the manufacturer under a push contract, and it is just the opposite under a pull contract. Generally speaking, the retailer’s profit under the pull contract is higher than the push contract. On the contrary, the manufacturer has higher profit under the push contract. Because the manufacturer bears the inventory risk under the pull contract, and the retailer assumes the inventory risk under the push contract.

Finally, the bank’s interest rate is affected by many factors, including the retailer’s credit guarantee level, the manufacturer’s capital status and the bank’s stock supervision cost. The results show that the bank will set higher interest rate to prevent and control risks under the pull contract. Under the pull contract, the manufacturer has to bear the cost of holding inventory and financing service, which increase the risk of bankruptcy.

This study contributes to the existing literature of SCF modeling by comparing the two different order contracts, push and pull, and focus on the financing mode of the retailer providing credit guarantee. Several contributions arise from this analysis. The results are of great value in guiding the retailer to manage different types of manufacturers, and can help the manufacturers with capital constraint decide how to finance. It is also beneficial for the bank to better control financing risk.

There exist some limitations in this paper and needs further exploration. Firstly, we assume that the bank is in a competitive credit market and sets interest rate only to meet break-even. While as a profit making organization, the bank will also be seeking to maximize their interests, which has an impact on the setting of interest rate. Secondly, this paper examines the supply chain composed of single retailer and manufacturer, while in fact there are multiple retailers and multiple manufacturers, which will need further exploration on the impact of the supply chain decision. Thirdly, there are many SCF modes except credit guarantee, and considering the impact of order contract mode under other financing modes will make the research more comprehensive. Finally, the study is carried out in a relatively ideal situation, but the information asymmetry in the supply chain, irregular demand distribution and the default risk not related to demand exist, thus exploring the influence of these factors on the supply chain decision will increase the practical significance of the paper.

Appendix A

Proof of Proposition 1.

Suppose that the order quantity is Q, $Q \le \frac{K_m}{C}$, the retailer first declares his potential maximal order quantity Q^M, and $Q = Q^M - \beta w$. Then acting as the follower, the manufacturer decides the wholesale price *w*. Against capital constraint, the manufacturer has the maximum production output of $\frac{K_m}{C}$, cannot meet the retailer's optimal order quantity.

(1) We first analyze the follower's optimal strategy. It is straightforward to derive that the manufacturer's profit function is concave in w from Eq. (5), as $\frac{d^2E[\pi_{m0}]}{dw^2} = -2\beta < 0$. Thus, for a given Q, let $\frac{dE[\pi_{m0}]}{dw} = Q^M - \beta w - \beta(w - C) = 0$, the manufacturer's best response can be derived as $w^* = \frac{Q^M}{2\beta} + \frac{C}{2} = \frac{Q}{\beta} + C$.

(2) Then we analyze the leader's optimal strategy. We substitute the response function w^* into Eq. (3), the retailer's profit can be given as $E[\pi_{r0}] = p\min\{Q, D\} - \mathrm{w}Q = pQ - p\int_0^Q F(D)dD - (\frac{Q^2}{\beta} + CQ)$, where $Q \le \frac{K_m}{C}$. It is obvious to derive that the retailer's profit function is concave in Q from Eq. (3), as $\frac{d^2E[\pi_{r0}]}{dQ^2} = -pf(Q) - \frac{2}{\beta} < 0$. Let $\frac{dE[\pi_{r0}]}{dQ} = p\bar{F}(Q) - \frac{2Q}{\beta} - C = 0$, we can get the optimal order quantity $Q^N = \frac{[p\bar{F}(Q^N) - C]\beta}{2}$ to maximize the retailer's profit.

However, as the manufacturer's capital is limited, the optimal order quantity cannot be reached, that is, $Q \le \frac{K_m}{C} < Q^N$, so we can get $\frac{dE[\pi_{r0}]}{dQ} > 0$. The retailer knows that there exists capital constraint, so the retailer will make the order of the manufacturer's largest production to maximize his profit. Then the optimal order quantity is $Q^* = \frac{K_m}{C}$, and we can get the optimal wholesale price $w^* = \frac{K_m}{C\beta} + C$.

Proof of Proposition 2.

As the manufacturer's revenue is wQ under the push contract, the manufacturer's liquid assets can fully repay the bank loan and the corresponding interest if $wQ - L(1+R_r) \ge 0$; otherwise, the retailer needs to pay part of the additional collateral costs, and the bank can only recover part of the loan. We can say that the manufacturer must be able to pay back the loan without bankruptcy risk, and the retailer doesn't need to repay the bank loan for the manufacturer.

(1) We first analyze the bank's interest rate. The bank is sure to take back all the loans and interest, so we can get $(CQ - K_m)R_r - Qh = (CQ - K_m)R_f$ from Eq. (1), then the bank's interest rate is $R_r^* = R_f + \frac{Qh}{CQ - K_m}$.

(2) We analyze the follower's optimal strategy. The manufacturer's profit function Eq.(4) can be simplified as $E[\pi_{m1}] = [w - C(1+R_f) - h](Q^M - \beta w) + K_m R_f$. It is straightforward to derive that the manufacturer's profit function is concave in w from Eq.(4), as $\frac{d^2 E[\pi_{m1}]}{dw^2} = -2\beta < 0$. Thus, for a given Q, the manufacturer's best response can be derived as $w^* = \frac{Q^M}{2\beta} + \frac{C(1+R_f)+h}{2} = \frac{Q}{\beta} + C(1+R_f) + h$, from $\frac{dE[\pi_{m1}]}{dw} = Q^M - \beta[2w - C(1+R_f) - h] = 0$.

(3) Finally, we analyze the leader's optimal strategy. We substitute the response function w^* into Eq. (2), the retailer's profit function can be given as $E[\pi_{r1}] = p\int_0^Q (D-Q)f(D)dD + (p - \frac{Q}{\beta} - C(1+R_f) - h)Q$, where $Q \le \frac{K_m}{C}$. It is obvious to derive that the retailer's profit function is concave in Q from Eq. (2), as $\frac{d^2E[\pi_{r1}]}{dQ^2} = -(\frac{pf(Q)}{4} + \frac{2\beta+1}{4\beta}) < 0$. We can get $Q_1 = \frac{p\bar{F}(Q_1) - C(1+R_f) - h}{(\frac{1}{\beta}+1)}$ from $\frac{dE[\pi_{r1}]}{dQ} = \frac{1}{2}[p\bar{F}(Q) - Q - \frac{Q}{\beta} - C(1+R_f) - h] = 0$. As the retailer's optimal order quantity Q^* should not be negative, then $Q^* = Q_1$ if $p\bar{F}(Q_1) > C(1+R_f) + h$; Otherwise $Q^* = 0$. Thus $Q^* = \max(Q_1, 0)$.

Proof of Proposition 3.

(1) We can obtain the retailer's profit under the push contract as:

$$E[\pi_{r0}] = p(\frac{K_m}{C} - \int_0^{\frac{K_m}{C}} F(D)dD) - \frac{K_m^2}{C^2\beta} - K_m,$$

$$E[\pi_{r1}] = (p - C(1+R_f) - h)Q^* - \frac{Q^{*2}}{\beta} - p\int_0^{Q^*} F(D)dD.$$

The retailer will participate in SCF if his expected profit under SCF is no less than the expected profit without SCF, that is, $E[\pi_{r1}] \ge E[\pi_{r2}]$. Then the sufficient condition for a retailer participating in SCF is the maximum manufacturer's initial capital level K_{m1}, where the threshold K_{m1} satisfies

$$E[\pi_{r1}] - E[\pi_{r0}] = (p - C(1+R_f) - h - \frac{Q^*}{\beta})Q^* + (1 + \frac{K_{m1}}{C^2\beta} - \frac{p}{C})K_{m1} + p\int_{Q^*}^{\frac{K_{m1}}{C}} F(D)dD \ge 0$$

(2) We can obtain the manufacturer's profit under the push contract as:

$$E[\pi_{m0}] = \frac{K_m^2}{C^2\beta},$$

$$E[\pi_{m1}] = \frac{Q^{*2}}{\beta} + K_m R_f \quad .$$

If the manufacturer is willing to participate in SCF, her profit in SCF is not less than the expected profit when not in financing, that is $E[\pi_{m1}] \geq E[\pi_{m0}]$. Then the sufficient condition for the manufacturer participating in SCF is the maximum manufacturer's initial capital level is K_{m2}, where the threshold K_{m2} satisfies

$$E[\pi_{m1}] - E[\pi_{m0}] = (Q^{*2} - \frac{K_{m2}^2}{C^2})\frac{1}{\beta} + K_{m2}R_f \geq 0$$

(3) The manufacturer's initial capital level has a maximum threshold $\min(K_{m1}, K_{m2})$ to make both the supply chain member to participate in the SCF service.

When the market demand distribution follows the uniform distribution with the upper bound of b and the lower bound of a, that is $f(D) = \frac{1}{b-a}$, $F(D) = \int_a^D \frac{1}{b-a} dD$, we can get $K_{m1} = \frac{M_0 - \sqrt{M_1 Q^{*2} + M_2 Q^* + M_3}}{2(b-a) + p\beta}$, $K_{m2} = \frac{C^2 \beta R_f - C\sqrt{C^2 \beta^2 R_f^2 + 4Q^{*2}}}{2}$,

where $M_0 = [(p-C)b + aC]\beta C$, $M_1 = [2(b-a) + p\beta](p+b-a)C^2$, $M_2 = -2[2(b-a) + p\beta][pb - (b-a)(C + CR_f + h)]\beta C^2$, $M_3 = [(C-p)b - aC]^2 C\beta^2$, $Q^* = \frac{pb\beta - \beta(b-a)(C + CR_f + h)}{(1+\beta)(b-a) + p\beta}$.

Proof of Proposition 4.

Under the pull contract, the retailer buys products after natural demand is realized, so his order quantity satisfies $Q = \min\{q, D\}$.

(1) We first analyze the follower's optimal strategy. It is straightforward to derive that the manufacturer's profit function is concave in q from Eq. (10), as

$\frac{d^2E[\pi_{m2}]}{dq^2} = -wf(q) < 0$. Thus, for a given Q, let $\frac{dE[\pi_{\mathrm{m}2}]}{dq} = w - C - wF(q) = 0$, the manufacturer's best response can be derived as $q^* = F^{-1}(\frac{w-C}{w})$.

(2) However, as capital constraint, the manufacturer's maximum production output satisfies $q \le \frac{K_m}{C} < F^{-1}(\frac{w-C}{w})$. In this case, $\frac{dE[\pi_{m2}]}{dq} > 0$, so the manufacturer's expected return increases with the production quantity, then the optimal production output is $q^* = \frac{K_{\mathrm{m}}}{C}$.

(3) Then we analyze the leader's optimal strategy. We substitute the response function q^* into $Q = \min\{q, D\}$, then we can get the optimal order quantity is $Q = \min\{\frac{K_m}{C}, D\}$.

Proof of Proposition 5.

The manufacturer promised a minimum yield $\frac{L(1+R_r)}{w}$ when applying for a loan with the retailer's credit guarantee. Therefore, the revenue of the manufacturer can fully repay the bank loan when the market demand is greater than the output. Thus, if the condition under which the manufacturer cannot repay the bank loan occurs, it must be that market demand is less than production output. In this case, the manufacturer's income is less than the amount of the loan, namely $wD < L(1+R_r)$. Therefore, the minimum demand should satisfies $\hat{D}_1 = \frac{L(1+R_{\mathrm{r}})}{w}$ to promise the manufacturers' no bankruptcy.

(1) We first analyze the bank's interest rate. We can get the bank's interest rate is $R_r^* = R_f + \frac{qh}{L}$ from Eq. (6).

(2) We then analyze the follower's optimal strategy. It is straightforward to derive that the manufacturer's profit function is concave in q from Eq.(9), as

$\frac{d^2E[\pi_{m3}]}{dq^2}=-wf(q)+\frac{h^2}{w}f(\hat{D}_1)<0$. Thus, for a given Q, the manufacturer's best response can be derived as $q^N=F^{-1}(1-\frac{C+h-hF(\hat{D}_1)}{w})$, from $\frac{dE[\pi_{m3}]}{dq}=w-C-wF(q)+hF(\hat{D}_1)-h=0$. However, q^N may exceed the manufacturer's constraint on the yield of $\frac{L+K_m}{C}$ with the variation of the distribution function, so the manufacturer's optimal response satisfies $q^*=\min(q^N,\frac{L+K_m}{C})$.

(3) Finally, we analyze the leader's optimal strategy. The retailer will bear some risk to provide credit guarantee, so the credit guarantee level will not exceed the amount of capital required by the manufacturer to produce. The optimal guarantee amount is to make the manufacturer just to provide production $\frac{L+K_m}{C}$.

We substitute the response function $q^*=\min(q^N,\frac{L+K_m}{C})$ into Eq. (7), and it is straightforward to derive that the retailer's profit function is concave in L, as $\frac{d^2E[\pi_{r3}]}{dL^2}=-\frac{p-w}{C^2}f(q^*)-\frac{(1+R_r^*-\frac{hK_m}{CL})^2}{w}f(\hat{D}_1)<0$.

- When $q^*=q^N$, the manufacturer can achieve optimal production without capital constraint. In this case, $\frac{dE[\pi_{r3}]}{dL}<0$, and the retailer's profits are decreasing with L. Then the retailer's optimal credit guarantee level L^* is just enable the manufacturer to achieve the optimal output, so $L^*=Cq^N-K_m$.

- When $q^* = \frac{L+K_{\mathrm{m}}}{C}$, we can get the retailer's optimal credit guarantee level $L^* = L_0$ from $\frac{dE[\pi_{r3}]}{dL} = \frac{p-w}{C}[1-F(q^*)] - (1+R_r^* - \frac{hK_m}{CL})F(\hat{D}_1) = 0$, where L_0 satisfies $\frac{p-w}{C}(1-F(\frac{L_0+K_m}{C})) = (1+R_r^* - \frac{hK_m}{CL_0})F(\hat{D}_1)$.

Then we can get the optimal credit guarantee level L^* satisfies $L^* = \min\{CF^{-1}(1-\frac{C+hF(\hat{D}_1)}{w}) - K_m, L_0\}$.

Proof of Proposition 6.

(1) We can obtain the retailer's profit under the pull contract as:

$$E[\pi_{r2}] = (p-w)(\frac{K_{\mathrm{m}}}{C} - \int_0^{\frac{K_{\mathrm{m}}}{C}} F(D)dD)$$

$$E[\pi_{r3}] = (p-w)(q - \int_0^q F(D)dD) - \int_0^{\frac{L(1+R_r)}{w}} wF(D)dD$$

If the retailer is willing to participate in SCF, his profit in SCF is not less than the expected profit when not in SCF, that is $E[\pi_{r3}] \geq E[\pi_{r2}]$. Then the sufficient condition for the retailer participating in SCF is the maximum manufacturer's initial capital level is $K_{\mathrm{m}3}$, where the threshold $K_{\mathrm{m}3}$ satisfies

$$E[\pi_{r3}] - E[\pi_{r2}] = (p-w)(\frac{L}{C} - \int_0^{\frac{L}{C}} F(D)dD) - \int_0^{\frac{L(1+R_f+\frac{h}{C})+\frac{K_{m3}h}{C}}{w}} wF(D)dD \geq 0$$

(2) We can obtain the manufacturer's profit under the pull contract as:

$$E[\pi_{\mathrm{m}2}] = w(q - \int_0^q F(D)dD) - Cq$$

$$E[\pi_{\mathrm{m}3}] = E[w\min(q,D) - LR_r - Cq]^+$$

If the manufacturer is willing to participate in SCF, her profit in SCF is not less than the expected profit when not in SCF, that is $E[\pi_{m3}] \geq E[\pi_{m2}]$. Then the sufficient condition for the manufacturer participating in SCF is the maximum manufacturer's initial capital level is $K_{\mathrm{m}4}$, where the threshold $K_{\mathrm{m}4}$ satisfies

$$E[\pi_{m3}]-E[\pi_{m2}]=(\frac{w-h}{C}-1-R_f)L-\frac{K_{m4}h}{C}-w[\int_{\frac{L(1+R_f+\frac{h}{C})+\frac{K_{m4}h}{C}}{w}}^{\frac{L+K_{m4}}{C}}F(D)dD-\int_0^{\frac{K_{m4}}{C}}F(D)dD]\geq 0$$

(3) The manufacturer's initial capital level has a maximum threshold $\min(K_{m3},K_{m4})$ to make both the two to participate in the SCF service.

When the market demand distribution follows the uniform distribution with the upper bound of b and the lower bound of a, that is $f(D)=\frac{1}{b-a}$, $F(D)=\int_a^D\frac{1}{b-a}dD$ we can get K_{m3} satisfies $\min\{K_1,K_2\}$, K_{m4} satisfies $\min\{K_3,K_4\}$,

K_1 is the root of $A_1K^2+B_1K+C_1=0$; K_2 is the root of $A_2K^2+B_2K+C_2=0$;

K_3 is the root of $A_3K^2+B_3K+C_3=0$; K_4 is the root of $A_4K^2+B_4K+C_4=0$;

$A_1=(2wCNX_2-1)h^2+(NC-1)wX_2^2$, $B_1=-2wC(NX_1+MX_2)-2wX_1X_2(NC-1)$,

$C_1=2wC(NX_1^2+MX_1+ah)-wX_1^2$;

$A_2=(2wCNX_4-1)h^2+(NC-1)wX_4^2$, $B_2=-2wC(NX_3+MX_4)-2wX_3X_4(NC-1)$,

$C_2=2wC(NX_3^2+MX_3+ah)-wX_3^2$;

$A_3=(N^2+1)w^2X_2^2+2(Nh-w)wX_2+h^2$,

$B_3=2(N-1)bCw^2X_2-2(N^2+1)w^2X_1X_2-2(Nh-w)wX_1-2bChw$,

$C_3=2(1-N)bCw^2X_1+(N^2+1)w^2X_1^2$;

$A_4=(N^2+1)w^2X_4^2+2(Nh-w)wX_4+h^2$,

$B_4=2(N-1)bCw^2X_4-2(N^2+1)w^2X_3X_4-2(Nh-w)wX_3-2bChw$,

$C_4=2(1-N)bCw^2X_3+(N^2+1)w^2X_3^2$;

$N=\frac{C(1+R_f)+h}{w}$, $M=(p-w)b+ac(1+R_f)+ah$, $X_1=\frac{Cw[(w-C)b+a(C+h)]}{w^2+h^2+C(1+R_f)h}$,

$$X_2 = \frac{Cw^2 + h^2}{w^2 + h^2 + C(1+R_f)h}, X_3 = \frac{MC}{p - w + N^2 w^2}, X_4 = \frac{Nwh^2}{p - w + N^2 w^2}.$$

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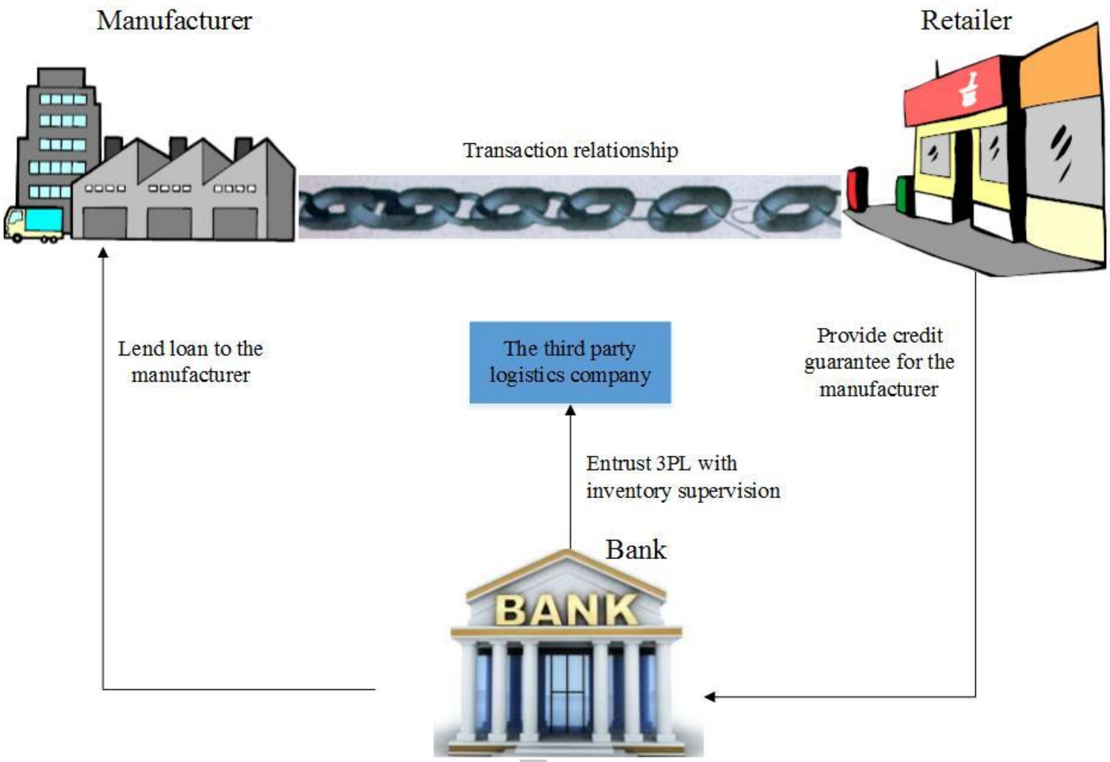
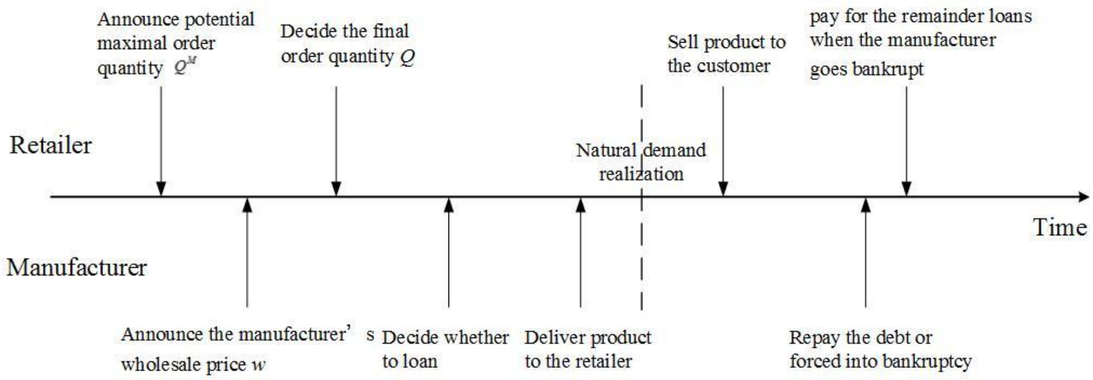
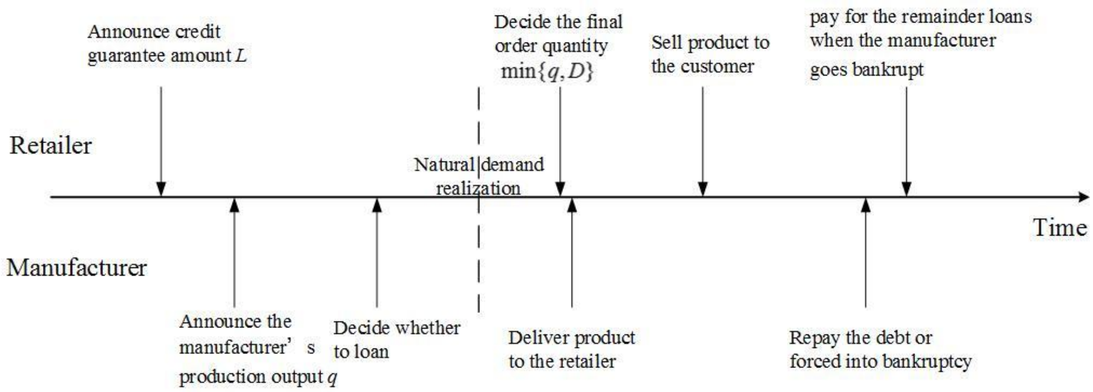
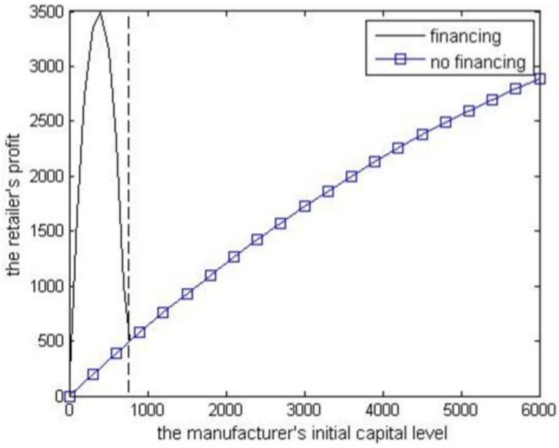
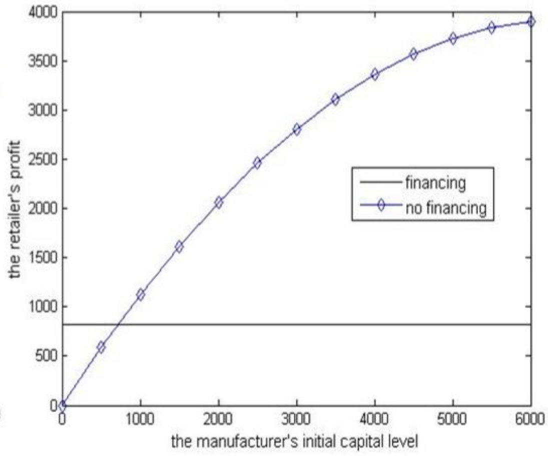

Manufacturer
Retailer
Transaction relationship
Lend loan to the manufacturer
The third party logistics company
Provide credit guarantee for the manufacturer
Entrust 3PL with inventory supervision
Bank
BANK


Announce potential maximal order quantity Q^M
Decide the final order quantity Q
Sell product to the customer
pay for the remainder loans when the manufacturer goes bankrupt
Retailer
Natural demand realization
Time
Manufacturer
Announce the manufacturer's wholesale price w
Decide whether to loan
Deliver product to the retailer
Repay the debt or forced into bankruptcy

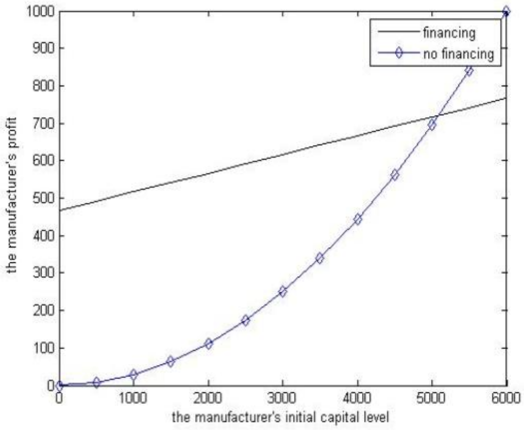

Announce credit guarantee amount L
Decide the final order quantity $\min\{q, D\}$
Sell product to the customer
pay for the remainder loans when the manufacturer goes bankrupt
Retailer
Natural demand realization
Time
Manufacturer
Announce the manufacturer's production output q
Decide whether to loan
Deliver product to the retailer
Repay the debt or forced into bankruptcy



Under the push contract



Under the pull contract



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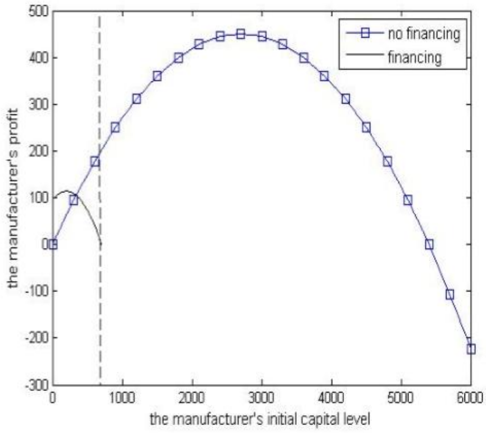

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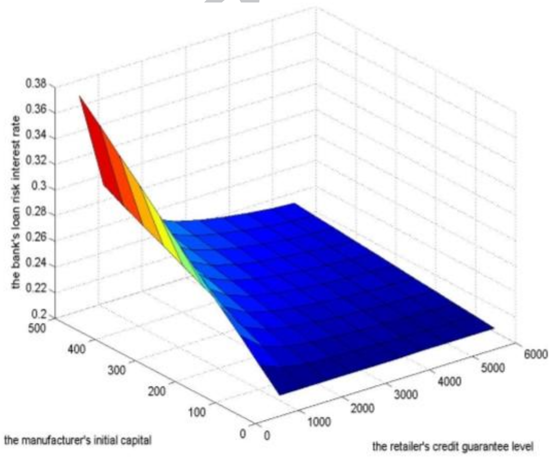

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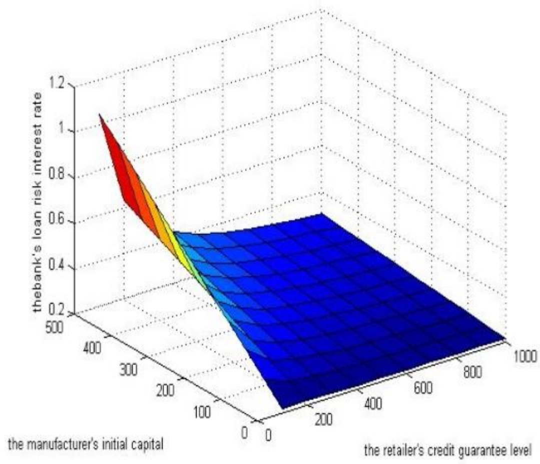

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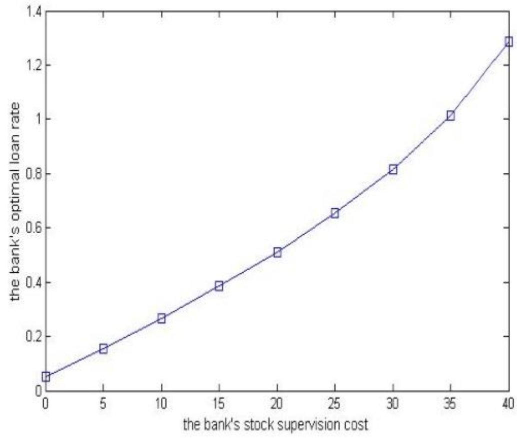

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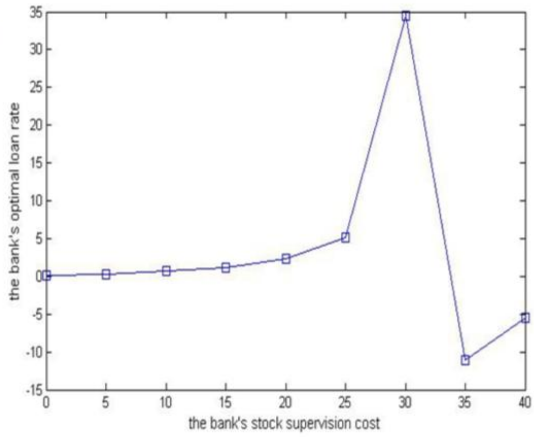

Under the pull contract